Chapter 2.6: Rational Functions and Their Graphs

Rational functions are quotients of polynomial functions. This means they are in fraction form where the denominator cannot be zero.

Find the Domain:

$$f\left(x\right)=\frac{x^2-9}{x-3}$$

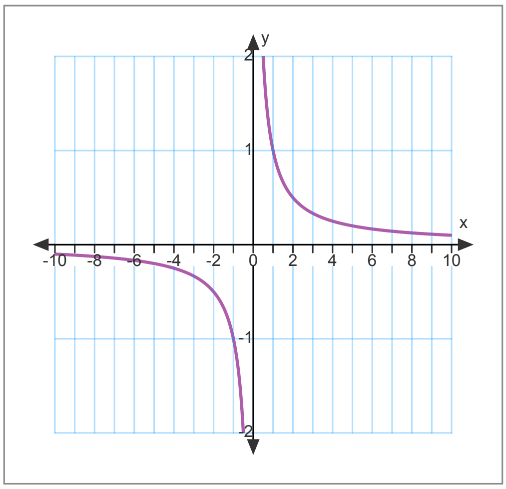
$$f\left(x\right)=\frac{x}{x^2-9}$$

$$f\left(x\right)=\frac{x+3}{x^2+9}$$

Parent Function:

$$f\left(x\right)=\frac{a}{x-h}+k$$

$$y=\frac{1}{x}$$



End Behaviors:

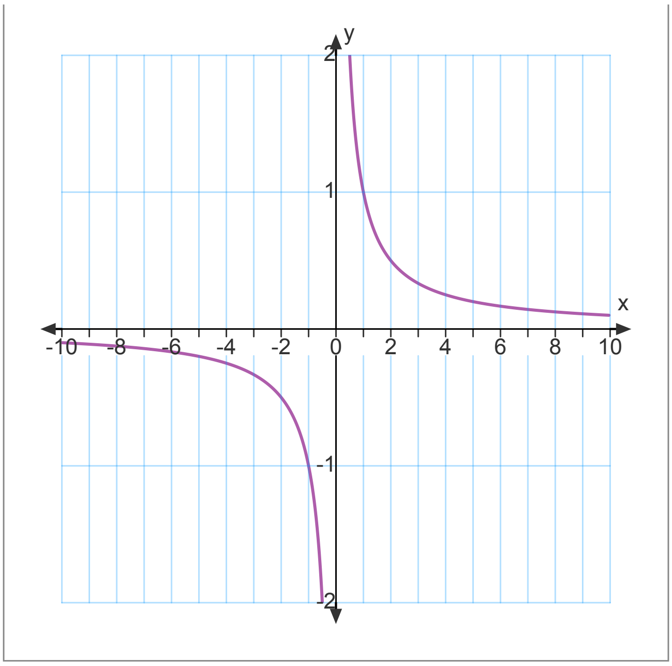
$$y=\frac{1}{x}$$

$x\rightarrow\infty$

$x\rightarrow-\infty$

$x\rightarrow0^{-}$

$x\rightarrow0^{+}$



Vertical Asymptotes:

a vertical line in which you domain is not
a member of.

The line x=a is a vertical asymptote of the graph pf a function f if f(x) increases or decreases without bound as x approaches a.

Picture

Find the vertical asymptotes of the functions:

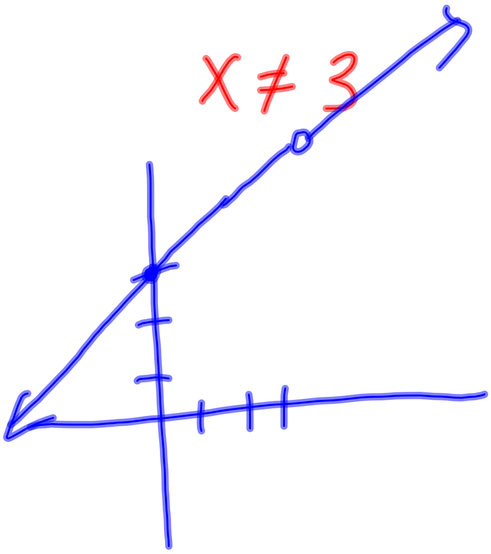
$$f(x)=\frac{x}{x^2-9} \qquad g(x)=\frac{x+3}{x^2-9} \qquad h(x)=\frac{x+3}{x^2+9}$$

Consider the equation: $f(x) = \left(\dfrac{x^2 - 9}{x - 3}\right)$

$$f(x) = \frac{(x+3)(x-3)}{x-3}$$

$$f(x) = x + 3$$

$x \neq 3$

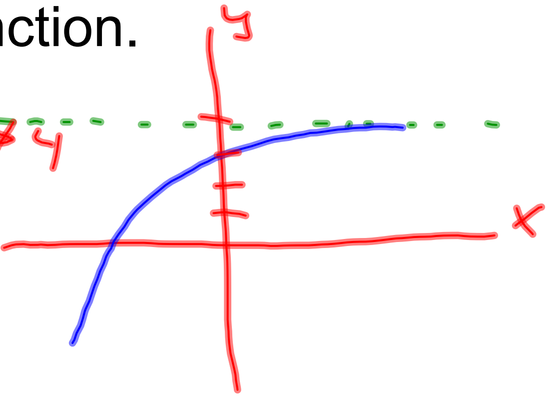


Horizontal Asymptotes:

The line y=b is a horizontal asymptote of the graph of a function f if f(x) approaches b as x increases or decreases without bound.

values that cannot be included in the range of a function.

$y \in \mathbb{R} \mid y \neq 4$



the graph has at most one horizontal determined but the degrees of p(x) and q(x)

↘top ↘bottom

if the degree of p(x)=m and q(x)=n

① m<n - asymptote y=0

② m=n - asymptote y=a/b leading terms of p(x)/q(x)

③ m>n no asymptote, end behavior

$$y=\frac{a}{b}x^{m-n}$$

Find the horizontal asymptotes

$$f(x)=\frac{4x}{2x^2+1} \qquad g(x)=\frac{4x^2}{2x^2+1} \qquad h(x)=\frac{4x^3}{(2x^2+1)}$$

$1<2$

$\boxed{y=0}$

$\boxed{y=2}$

$\boxed{\text{none}}$

Strategy for Graphing a Rational Function:

$$f(x)=\frac{p(x)}{q(x)}$$

1. Determine the symmetry
2. Find the y-int by f(0)
3. Find the x-int by f(x)=0
4. Find any vertical asymptotes
5. Find the horizontal asymptotes
6. Plot other points
7. Graph

$f(-x) = f(x)$ y-axis

$f(-x) = -f(x)$ orig.

Graph: $y=\frac{2x}{x-1}$

$\frac{2(-1)}{-1-1} = \frac{-2}{-2} = 1$

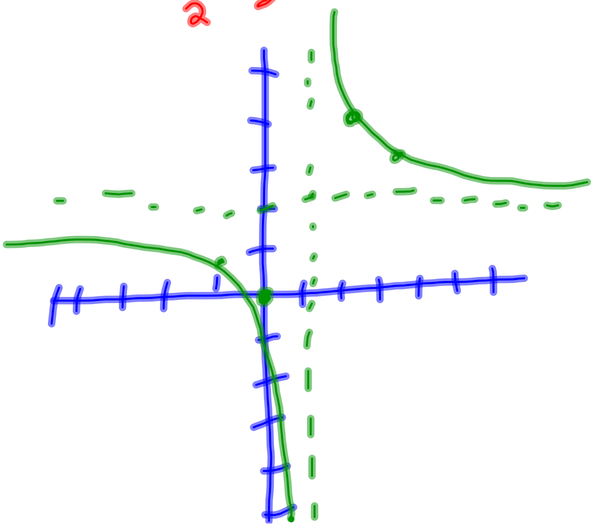
$\frac{2(2)}{2-1} = \frac{4}{1}$

y-int: $\frac{2(0)}{0-1} = \frac{0}{-1} = 0$ $(0,0)$

x-int: $(x-1)\,0 = \frac{2x}{x-1}\,(x-1)$ $\frac{0}{2} = \frac{2x}{2}$ $x=0$ $(0,0)$

Vert asy: $x=1$

horz asy: $y=2$



Graph: $y = \frac{3x^2}{x^2 - 4}$

Graph: $y = \frac{2x^2}{x^2 - 9}$

Slant Asymptotes:

Has a slant asymptote if the degree of the numerator is greater than the degree of the denominator.

$$A(x)=\frac{x^2+1}{x-1}$$

$$\begin{array}{r|rrr} 1 & 1 & 0 & 1 \\ & & 1 & 1 \\ \hline & 1 & 1 & \boxed{2} \end{array}$$

$$\begin{array}{r} \boxed{x+1} + \frac{2}{x-1} \\ x-1\,\big)\overline{\cancel{x^2} + 0x + 1} \\ \ominus\,\cancel{x^2} - x \\ \hline x + 1 \\ \ominus\,\cancel{x} - 1 \\ \hline 2 \end{array}$$

slant asy.

Find the slant asymptote:

$$f(x)=\frac{(x^2-4x-5)}{(x-3)}$$

slant asy

$$\begin{array}{r} \boxed{x-1} + \frac{-8}{x-3} \\ x-3\,\big)\overline{x^2 - 4x - 5} \\ \ominus\,x^2 - 3x \\ \hline -\cancel{x} - 5 \\ \ominus\,-\cancel{x} + 3 \\ \hline -8 \end{array}$$

A company is planning to manufacture wheelchairs that are light, fast and beautiful. Fixed monthly cost will be $500,000 and it will cost $400 to produce each radically innovative chair.

Write the cost function, C, of producing x wheelchairs.

Write the average cost function, $\overline{C}$, of producing x wheelchairs

Find and interpret $\overline{C}(1,000)$, $\overline{C}(10,000)$ $\overline{C}(100,000)$

What is the horizontal asymptote for the average cost function, $\overline{C}$? Describe what this represents for the company.

Two commuters drove to work a distance of 40 miles and then returned again on the same route. The average velocity on the return trip was 30 miler per hour faster than the average velocity on the outgoing trip. Express the total time required to complete the round trip, T, as a function of the average velocity on the outgoing trip. (d=vt)

Suggested Homework: Chapter 2.6
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